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Consequences of Flavor as a Dynamical Quantum Number

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ABSTRACT

Phenomenological consequences of composite leptons and quarks are studied in the class of models in which the fermions of the first generation (i.e. e, ν_e, u and d) are the ground states of (unspecified) composite systems and each new generation is a radial excitation level. We find that the standard QED results remain practically unaffected in such a scheme. The excited quarks/leptons having mass larger than 5 GeV could be produced in e^+e^- experiments and would increase the value of R by several per cent as compared with the case of the pointlike quarks and leptons. An argument is given for the smallness of the anomalous magnetic moments of composite leptons.

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Due to the proliferation in the number of quarks and leptons there has been recently a revival of interest in composite models for these particles. Parallel with this, a number of objections against such models have been raised (see e.g.1,2). These are essentially based on the agreement of QED with experiment and the assertion that the existence of lepton substructure and/or excited lepton states would affect several well established QED results, in particular the value of the anomalous magnetic moment of leptons. In this Letter we remove these objections and formulate some phenomenological consequences of the compositness which may be tested in experiment.

As simplest and probably the most economical class of models we propose to consider the one in which the ordinary leptons and quarks (i.e. ν_e,e,u,d) are the ground states, in particlar the $1S_{1/2}$ states of (presently unspecified) composite systems and each new generation is an excitation level of it. The facts that all quarks have the same parity and that there are no significant electromagnetic transitions between different generations suggest that this be only the radial excitation. Thus muon is the $2S_{1/2}$ state of the same system as electron, etc. In order to make this scheme compatible with the data we have to assume that the $L>0$ states are for some reason much heavier than the observed fermions.

The breakdown of QED in this scheme might occur via coupling of leptons to various excited states. The dominant contributions are expected to come from the electric dipole (E1) coupling between states of opposite parities and the magnetic dipole (M1) coupling between states of same parity. We discuss briefly several consequences of these couplings. More numerous and detailed calculations will be published elsewhere.[3]

I. ELECTRIC COUPLING

The phenomenological Lagrangian for the E1 coupling of a quark/lepton state (q,ℓ) to a spin 1/2, L=1 state (Q,L) of mass M can be set up as

$$\mathcal{L}_E = \frac{d}{2}\,\overline{\psi}_\ell\gamma_5\sigma_{\mu\nu}\psi_L F^{\mu\nu} + \text{H.c.}, \qquad (1)$$

where d is a phenomenological parameter with the meaning of the transition electric dipole moment. At energies much smaller than M we can approximate the propagator of the P state by 1/M and the only free parameter is the ratio d^2/M.

A. The Decay $\mu\to e\gamma\gamma$.

The natural wav for this decay to occur is via double E1 transition with an intermediate P state (Fig. 1). The total decay rate is

$$\Gamma_{\mu\to e\gamma\gamma} \approx \left(\frac{d^2}{M}\right)^2 \frac{m_\mu^7}{14\cdot 10^3\pi^3} \quad . \tag{2}$$

For the branching ratio of this decay we thus have

$$BR(\mu\to e\gamma\gamma) \approx \left(\frac{d^2}{M}\right)^2 \frac{m_\mu^2}{64G_F^2} \quad .$$

From the experimental upper bound $BR(\mu\to e\gamma\gamma)<4\cdot 10^{-6}$ we obtain the upper bound for the parameter d^2/M:

$$\frac{d^2}{M} < 16\cdot 10^{-7}\ \mathrm{GeV}^{-3} \quad . \tag{3}$$

From this we can roughly estimate the order of magnitude of the mass M: $d \backsim e\cdot 10^{-16}$cm leads to the lower bound

$$M > 5\ \mathrm{GeV} \quad . \tag{4}$$

We remark that both experimental inputs ($BR(\mu\to e\gamma\gamma) < 4\cdot 10^{-6}$ and $r_0 < 10^{-16}$cm) used to derive (4) are <u>upper bounds</u> and may be improved by more precise measurements. However, this need not necessarily change dramatically the order-of-magnitude estimate (4) of the mass M, since the two improvements would tend to change M in opposite directions.

B. Compton Scattering on Electrons

The contribution from the graphs of Fig. 2 to the differential cross section averaged over polarizations is computed to be (in the rest frame of the electron):

$$\frac{d\bar{\sigma}_L}{d\Omega_{k'}} = \frac{1}{2\pi^2}\left(\frac{d^2}{M}\right)^2 \frac{1}{m_e^2}\, kk'^3 \sin^4\frac{\theta}{2}\left\{m_e^2\left(1+\frac{k'^2}{k^2}\right)\left(1+\cos^2\frac{\theta}{2}\right)+\right.$$

$$+\,4m_e^2\,\frac{k'}{k}\cos\theta\cos^2\frac{\theta}{2} - 4\,k'^2\sin^6\frac{\theta}{2}\cos^2\frac{\theta}{2} +$$

$$\left. +\,2m_e\,k'\left(1-\frac{k'}{k}\right)\cos^2\frac{\theta}{2}\right\},$$

with

$$k' = \frac{k}{1+\frac{k}{m_e}\;(1+\cos\theta)},\quad k \ll \frac{M^2}{m_e}\;.$$

The cross section reaches its maximal value at $\theta_m \approx 2\sqrt{\frac{m_e}{k}}$. The integration over θ yields the following contribution to the total cross section:

$$\sigma_L \approx \frac{1}{4}\,10^{-3}\left(\frac{d^2}{M}\right)^2 m_e^{3/2}\,k^{5/2}\;. \tag{5}$$

Comparing this with the first-order QED result

$$\sigma_{QED} = \frac{\pi\alpha^2}{km_e}\left(\ln\frac{2k}{m_e} + \frac{1}{2}\right) , \tag{6}$$

and making use of the bound (3), we see that the contribution from the excited-lepton intermediate state is many orders of magnitude smaller:

$$\frac{\sigma_L}{\sigma_{QED}} < 10^{-20}\left(\frac{k}{m_p}\right)^{7/2}\frac{1}{\ln\frac{2k}{m_e}} .$$

Although showing that the QED computation remains unaffected,† this result does not imply that the excited lepton states are completely invisible. Near the resonance the total cross section for the Compton scattering on electrons can be written in the Breit-Wigner form

$$\sigma_L = \frac{\pi}{k^2}\frac{\Gamma^2}{(E-M)^2+\Gamma^2/4} ,$$

where $\Gamma = d^2M^3/8\pi$. Using the same values for d and M as before, we estimate the width of the excited lepton states $\Gamma \approx (0.1 \div 10)$MeV for $M = (5\div 20)$GeV. On the top of the resonance the cross section is

$$\sigma_L \Big|_{k\approx M/2} = \frac{16\pi}{M^2} ,$$

and is sizable fraction of the QED cross section (6):

$$\frac{\sigma_L}{\sigma_{QED}} \approx 10^4 \frac{m_e}{M} \approx \frac{5}{M(\mathrm{GeV})} \;.$$

However, since the resonance formation in the Compton scattering on the stationary electrons requires extremely high photon energies, one might look for the excited leptons in $e^+e^- \to Le$ or $L\mu$ (Fig. 3). The cross section for this process as compared to the standard QED process $e^+e^- \to \mu^+\mu^-$ is of the order d^2s/α which amounts to a few up to 15 percent at $\sqrt{s} \gtrsim M+m_e$ or $M+m_\mu$. The resonance L (e^*, μ^* or τ^*) decays predominantly into an ordinary lepton and a photon, with the width Γ given before. The signature for the production of L would thus be a sequence of peaks (or, possibly, a broad enhancement) in the invariant mass distribution of $e\gamma$ or $\mu\gamma$ in the final state. Concluding, we note that in the case of the composite leptons and quarks the predicted values of R at the energies $\sqrt{s} \gtrsim M$ are a few up to 15 per cent larger than in the case of the pointlike ones.

Another simple reaction to produce the L would be[‡]

$$e + p \to L + \text{anything}$$
$$\hookrightarrow e + \gamma \text{ or } \mu + \gamma \;.$$

The L would again be observed in the invariant mass plot of the final $e+\gamma$ or $\mu+\gamma$.

II. MAGNETIC COUPLING AND THE ANOMALOUS MAGNETIC MOMENTS

The processes like $\mu\to e\gamma$ are in this scheme due to the magnetic coupling of the form

$$\mathcal{L}_m = \frac{\lambda}{2}\,\bar{\psi}_\mu \sigma_{\mu\nu}\psi_e F^{\mu\nu} + \text{H.c.}, \tag{7}$$

where λ has the meaning of the transition magnetic dipole moment. The total width of the muon decay into an electron and a photon can be computed to be

$$\Gamma = \frac{\lambda^2 m_\mu^3}{8\pi},$$

which leads to the branching ratio

$$\mathrm{BR}(\mu\to e\gamma) \approx \frac{200\lambda^2}{G_F^2 m_\mu^2}.$$

The present experimental upper bound for this branching ratio is[5] $1.9\cdot 10^{-10}$ and we derive the upper bound for λ:

$$\lambda < 10^{-12}\ \text{GeV}^{-1}.$$

In principle the Lagrangian (7) could contain the diagonal (Pauli) terms

$$\mathcal{L} = \frac{e\kappa_e}{2m_e}\,\bar{\psi}_e\sigma_{\mu\nu}\psi_e F^{\mu\nu} + \frac{e\kappa_\mu}{2m_\mu}\,\bar{\psi}_\mu\sigma_{\mu\nu}\psi_\mu\, F^{\mu\nu} + \ldots \tag{8}$$

with $\kappa_e, \kappa_\mu \ldots$ being the anomalous magnetic moments of leptons, $\kappa_i = (g_i - 2)/2$. It has often been remarked (see e.g. 2) that, in general, there is no reason for a composite system to have small anomalous magnetic moment. We comment briefly on this point.

The magnetic moment of <u>any</u> system of total spin 1/2, charge e and mass m can be written as $\mu = \mu_D(1+\kappa)$ where $\mu_D = e/2m$ is the Dirac moment and κ the anomalous part. For a composite lepton of the size of the bound state a and the mass m, κ must be a function of the ratio a/λ_c, $\lambda_c = m^{-1}$. The requirement of the renormalizability of QED in the case of pointlike leptons implies $\kappa(0)=0$. For $a \neq 0$ QED is, of course, not renormalizable: it becomes an effective low-energy theory with the cutoff $1/a$. If the limit $a \to 0$ is smooth we can write

$$\kappa \approx C\,\frac{a}{\lambda_c}\,, \tag{9}$$

where the constant C depends on the coupling constant of the underlying theory. The formula (9) suggests the ratio of the size and the Compton wavelength of an object as a measure of its anomalous magnetic moment. The anomalous magnetic moments of leptons have to be small, because they are intrinsically small ($a \ll \lambda\!\!\!^{-}_c$) objects. Thus the problem of constructing the magnetic moments of composite leptons, as mentioned in Ref. 2, appears to be automatically solved by constructing the bound state of appropriately small size.[§]

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FOOTNOTES

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†This is also true for several other QED processes which we have computed.[3]

‡The search for excited leptons and, in particular, this reaction have been proposed by F. Low.[4]

§Composite models for leptons which produce small anomalous magnetic moments have also been investigated by S. Brodsky and S. Drell (in preparation). (S. Brodsky, private communication.)

REFERENCES

1. M. Glück. Phys. Lett. 87B, 247 (1979).
2. H. Lipkin, Preprint FERMILAB-Conf-79/60-THY.
3. V. Višnjić-Triantafillou, in preparation.
4. F.E. Low. Phys. Rev. Lett. 14, 238 (1965).
5. J.D. Bowman *et al.*, Phys. Rev. Lett. 42, 556 (1979).

FIGURE CAPTIONS

Fig. 1 The decay $\mu \rightarrow e\gamma\gamma$ as a double E1 transition via an excited-lepton intermediate state L.

Fig. 2 The contribution of the excited lepton intermediate state to the Compton scattering on electrons.

Fig. 3 The diagrams for the production of L in the e^+e^- scattering. The excited lepton L decays subsequently in $e\gamma$ or $\mu\gamma$.

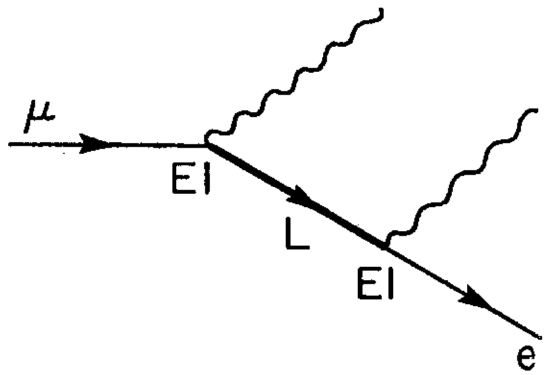

Fig. 1

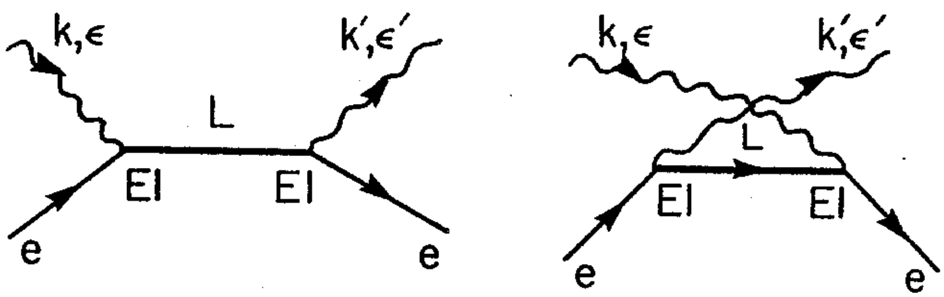

Fig. 2

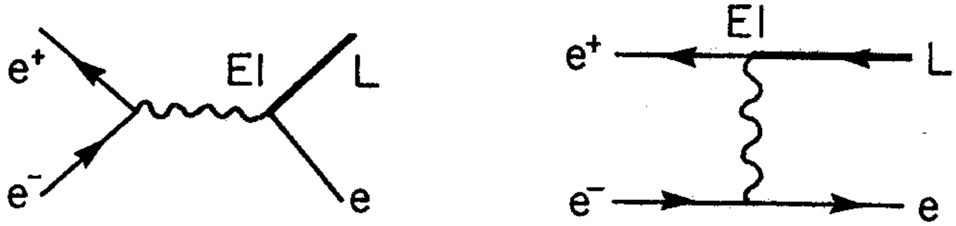

Fig. 3